

GEOMETRIC BROWNIAN MOTION IN ANALYZING SEASONALITY OF GOLD DERIVATIVE PRICES

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ABSTRACT

Complex financial markets, influenced by complex and interconnected factors, require proper decision making. Gold derivatives, as an increasingly popular trading instrument, have experienced significant growth. However, with high profit potential also comes significant risk. Market analysis, including technical analysis and leveraging seasonality, can be an important tool in reducing risk and making smart decisions. This research aims to assess the effectiveness of the Geometric Brownian Motion (GBM) model in predicting Gold Derivative prices through the application of the Mean Absolute Percentage Error (MAPE) test. In this study, the Brownian Motion Geometric Method and Simple Moving Average are combined to analyze the seasonality of gold derivative prices to provide a view of price movement patterns. The results showed that the Brownian Motion Geometric Method was effective in predicting the price of gold derivatives, with a low error rate. In addition, seasonality analysis reveals monthly price movement patterns that can be a guide for traders and investors. This research provides valuable insights for decision making in gold derivatives trading in dynamic and complex financial markets.

KEYWORDS Financial market; gold price derivative; geometric Brownian motion (GBM); Simple Moving Average (SMA); Mean Absolute Percentage Error (MAPE); Seasonality Analysis



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INTRODUCTION

In the financial market, there is a complex system that is influenced by various factors that are interconnected and not linear. Therefore, understanding when and how to make decisions in trading this market is very significant (Naranjo et al., 2018). Currently, many investors and traders are interested in derivative trading instruments. This instrument

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is a financial agreement between two or more parties that aims to fulfill an agreement in buying or selling assets or commodities (Gunarsa, 2019).

According to Lumintang (2020), investor interest in derivatives is no longer taboo, as can be seen from the daily trading activities on the futures market. This market has continued to grow from \$1.2 trillion in 2001 to \$6.6 trillion in 2019. This growth has also continued with daily volumes reaching \$5.3 trillion in the United States, making it the largest futures market on a financial exchange. One of the popular alternative trading products among market participants today is gold derivatives.

Gold derivative trading is included in the Foreign Exchange Market (forex) product category. Investment in forex trading is a very profitable business because of the high rate of return. In the forex market, the amount of money in circulation reaches 3.8 trillion USD every day. High price volatility and a large level of liquidity make forex investment a commodity that has the potential to earn large and fast profits. However, the promised profit level is proportional to the existing risk level (Pilliangsani, 2012).

Significant risks can be mitigated through market analysis. Analysis is the process of calculating, evaluating, and measuring past and current events or data to forecast future price direction. In the world of trading, there are two types of analysis that are commonly used, namely technical analysis and fundamental analysis. Fundamental analysis is based on global economic, political, and security factors, while technical analysis focuses more on market movements themselves. Therefore, it is important to have references such as seasonal trends in price movements known as seasonality to help get good results from technical and fundamental analysis. Seasonality describes the patterns of price movements that occur in the forex and commodity markets, also known as seasonal movements. This pattern shows how prices move based on predictable trends over certain periods of the year (Girardin & Namin, 2019).

The Geometric Brownian Motion method is a mathematical method commonly used to predict asset prices within a certain period. In this study, Geometric Brownian Motion will be applied to predict the price of gold derivatives which have a higher level of volatility, liquidity and fluctuations than stocks. Besides that, the Simple Moving Average (SMA) will also be used by looking at the trend pattern for each period resulting from the calculation of the moving average. This research will test the feasibility of the Geometric Brownian Motion Method in predicting gold derivative prices by testing accuracy using the Mean Absolute Percentage Error (MAPE) and analyzing trend movement patterns using the Simple Moving Average. Furthermore, The combination of the two methods will be analyzed for seasonality to determine the picture of gold derivative price movements for the next period. Value at Risk level analysis will also be carried out to estimate the level of risk with the help of Monte Carlo Simulation.

Research on predicting gold prices has been carried out by several researchers such as Lawrence and Mallikarjun (2022) who predict annual gold prices using the Autoregressive Distribution Lag (ARDL) model using the level of demand for gold, interest rates on securities, and historical prices. Ibrahim et al (2021) conducted research involving Geometric Brownian Motion (GBM) and Geometric Fractional Brownian Motion (GFBM) involving the price of Malaysian crude palm oil, the results showed that the GBM model is more suitable for identifying phenomena that are not long memory. The Moving Average method was used in research conducted by Kuo, et al (2021) which shows that the proposed system flexibly finds better trading points and outperforms traditional methods and generates significant profits in both emerging and emerging stock markets. This study aims to analyze the seasonality of gold derivatives by combining the Geometry Brownian Motion (GBM) model in predicting Gold Derivative prices and analyzing trend

patterns formed using the Simple Moving Average (SMA) method. The results of the combination test between these mathematical theories will be used as a reference in determining the trend seasonality of gold derivative price movements. The seasonality of price movement trends can be used as a description and reference for traders and investors in conducting technical and fundamental analysis.

This research aims to assess the effectiveness of the Geometric Brownian Motion (GBM) model in predicting Gold Derivative prices through the application of the Mean Absolute Percentage Error (MAPE) test. The obtained test results were used to determine the Seasonality trend, offering traders and investors valuable insights for conducting technical and fundamental analyses.

Return

Return asset or is the rate of return or profit generated from an investment in a financial asset. Return for a period $(R)t_i$ with is defined as the logarithmic comparison of the asset price and the asset price in the previous period. Mathematically, it is formulated as follows ($i = 1,2,3, \dots, n$) $S(t_i)S(t_i)$ (Ruppert & Matteson, 2011):

$$R(t_i) = \ln \left(\frac{S(t_i)}{S(t_{i-1})} \right) \tag{2.1}$$

Normal Distribution

The normal distribution is the level of data distribution that collects around the mean (mean) and spreads to the right and left with a certain standard deviation. The mean determines the location of the center of the curve, while the standard deviation controls how wide or narrow the data spreads around the mean.

A random variable X with the opportunity density function can be expressed as a normal distribution with parameters and if it satisfies the following equation $(f(x))\mu\sigma$ (Navidi, 2006):

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left[-\frac{1}{2} \left(\frac{x - \mu}{\sigma} \right)^2 \right] \tag{2.2}$$

for μ , σ , and x . Mathematically, it is written as $-\infty < x < \infty - \infty < \mu < \infty \sigma > 0$
 $X \sim N(\mu, \sigma^2)$

Log-Normal Distribution

The normal distribution is generally not suitable for data with high skewness or outliers. The log-normal distribution, which is a subset of the normal distribution, is often a good choice for such data sets (Navidi, 2006):

The relationship between normal and log-normal distribution can be stated as follows:

- If $X \sim N(\mu, \sigma^2)$, then the random variable is log-normally distributed with parameters and $Y = e^X \mu \sigma^2$
- If Y is log-normal distribution with parameters μ and σ^2 , then the random variable $X = \ln Y$ is normally distributed $N(\mu, \sigma^2)$.

The probability density function of a log-normal random variable with parameters and is: $\mu\sigma$

$$f(y) = \begin{cases} \frac{1}{y\sigma\sqrt{2\pi}} \exp \left[-\frac{(\ln(y) - \mu)^2}{2\sigma^2} \right], & \text{jika } y > 0 \\ 0, & \text{jika } y \leq 0 \end{cases} \tag{2.3}$$

Kolmogorov–Smirnov Normality Test

The Kolmogorov-Smirnov normality test is a statistical method used to test whether a data sample comes from a normal distribution or not. This test was developed by mathematicians named Andrey Kolmogorov and Nikolai Smirnov, whose aim is to determine the degree to which data fits a normal distribution (Siegel et al., 1997). If a function with a cumulative distribution is normally distributed F_z with empirical cumulative probability, then pF_k Testing the normality of the data using the Kolmogorov-Smirnov test procedure is as follows (Daniel, 1989):

1. Determine the hypothesis H_0 And H_1
 $H_0: F_k = F_z$ (Sample data comes from normal distribution).
 $H_1: F_k \neq F_z$ (Sample data is not from a normal distribution).
2. Set a significance level of α
3. Test statistics:

$$D = \max|F_k - F_z|$$

Rejection criteria

H_0 rejected if the value of D is greater than the quantile of the Kolmogorov-Smirnov test table (α), or the value of $.1 - \alpha D_{tabel} p - value < \alpha$

Volatility

Given a logarithmic series return $R(t_i)$ for with in the Geometric Brownian Motion model, it is necessary to estimate the sample mean and the respective volatility as follows ($t_i = 1, 2, 3, \dots, n$) (μ) (σ^2) (Di Asih & Purbowati, 2009):

$$\mu = \frac{1}{n} \sum_{i=1}^n R(t_i) \quad (2.4)$$

And

$$\sigma^2 = \frac{1}{n-1} \sum_{i=1}^n (R(t_i) - \mu)^2, \quad (2.5)$$

Stochastic Process

The stochastic process $\{W, t, t \in T\}$ consists of a group of random variables, where each $W(t)$ is a random variable. The set T is the set of indices that represent time. If T can be calculated, then stochastic processes are categorized as discrete-time processes. In this case, T can be expressed as the set $T = \{t-1, t-2, \dots, t-n\}$ where $t-i$ is a quantifiable value of time, such as a non-negative integer. If T consists of uncountable sets, then stochastic processes are categorized as continuous-time processes, suppose T is an infinite set $[0, \infty)$, then $W(t)$ for $t \geq 0$ is a random variable representing continuous-time stochastic processes (Ross, 2014).

Theorem $It\hat{o}$

In general, in finance with a continuous time model, it is often assumed to be an $It\hat{o}$. Suppose there is a continuous function $F(P_t, t)$ which depends on the variables P_t and t , where P_t is an $It\hat{o}$ process. which satisfies the stochastic differential equation (2.8). The μ and σ values in the stochastic differential equation are functions of the parameters P_t and t , while W_t is Brownian Motion (Wiener process). With this assumption, the general equation of the theorem $It\hat{o}$ are as follows (Hull, 2009):

$$F(P_t, t) = \left(\frac{\partial F(P_t, t)}{\partial P_t} \mu(P_t, t) + \frac{\partial F(P_t, t)}{\partial t} + \frac{1}{2} \frac{\partial^2 F(P_t, t)}{\partial P_t^2} \sigma(P_t, t)^2 \right) dt + \left(\frac{\partial F(P_t, t)}{\partial P_t} \sigma(P_t, t) \right) dW_t \tag{2.9}$$

Equation (2.9) is referred to as a formula *Itô*

Theorems *Itô* become important tools in financial analysis and modeling involving stochastic processes and continuous time processes, and make it possible to calculate the expected values of process-dependent functions and Brownian Motion.

Geometric Brownian Motion

Geometric Brownian Motion or also known as the Wiener process, is a stochastic process that has a continuous nature. Brownian motion is formed from the symmetrical random walk equation $W(t)$ by finding the limits of the random walk distribution. A stochastic process $B_t, t \geq 0$ is called Geometric Brownian Motion if it fulfills the following three conditions (Dmouj, 2006):

- i. $W(0) = 0$, with a probability of 1.
- ii. Each change is normally distributed, which means it has a mean of 0 and a standard deviation of 1. $W_t \sim N(0, t)$
- iii. For independent random variables. $t_1 \leq t_2 \leq t_3 \leq t_4 \leq \dots \leq t_n, W_{t_2} - W_{t_1}, W_{t_3} - W_{t_2}, W_{t_4} - W_{t_3}, \dots, W_{t_n} - W_{t_{n-1}}$

In general, the GBM model is expressed through equation (2.8) as follows:

$$dP_t = \mu(P_t, t)dt + \sigma(P_t, t)dW_t$$

for $t \in [0, T]$, term is the drift term $\mu(P_t, t)dt$, where $\mu(P_t, t)$ is the drift coefficient. While Sedangkan $\sigma(P_t, t)dW_t$ is the diffusion term, with $\sigma(P_t, t)$ is the diffusion coefficient. $W(t)$ is *Brownian Motion* Standard.

Mean Absolute Percentage Error (MAPE)

Suppose there are two data sets with periods t_i For $i = 1, 2, 3, \dots, n$ that is actual (Y_t) and predictive data, each of which consists of observations, then (F_t) *n Mean Absolute Percentage Error (MAPE)* with the following equation (Maricar, 2019):

$$MAPE = \frac{\sum_{p=1}^n \left| \frac{Y_p - F_p}{Y_p} \right|}{n} \times 100\% \tag{2.15}$$

The level of prediction accuracy scale can be concluded based on the following summary table:

MAPE value	Forecasting Accuracy
<10%	Excellent accuracy
11% - 20%	Good accuracy
21% - 50%	Accuracy within reasonable limits
>51%	Accuracy is not accurate

Source: Lawrence, K D., Klimberg RK, & Lawrence S. M

Simple Moving Averages

Suppose a series of data series consisting of *t* observation, that is. To calculate the Simple Moving Average with a period is as follows (X_1, X_2, \dots, X_n) (Kuo & Chou, 2021):

$$SMA(n) = \frac{P_{t-n+1} + P_{t-n+2} + \dots + P_t}{n} \quad (2.16)$$

So that equation (2.16) can be simplified to:

$$SMA(n) = \frac{1}{n} \sum_{t=n+1}^n P_t \quad (2.17)$$

The use of the Simple Moving Average to read trends can be seen in the following table.

Table 2. Table of Simple Moving Average (SMA) conditions.

No	High school position	Meaning	Type
1	SMA is below the price	Uptrend / Uptrend	<i>Trends</i>
	The SMA is above the price	Trend Down / Downtrend	
2	SMA cuts the price from above the price or vice versa	Reversal / reversal of direction	<i>Reversal</i>
3	The price moves up and down crossing the SMA	Rank	<i>Rating</i>

Source: Achelis, S.B

Seasonality

Seasonality in the financial market refers to the tendency found in the foreign exchange (forex) market where there is a pattern that repeats over a certain period of time. These patterns can be related to changes in trading volume, volatility, or the direction of price movements over a certain period of time in a year (Girardin & Namin, 2019). Seasonality refers to certain times of the year marked by changes, which may impact economic, political or business aspects. According to Fountas et al (2016), all physical activity, including activity in the forex and commodity fields, is influenced by seasonality.

RESEARCH METHOD

The data used in this study is historical data from the closing price of gold derivatives for a period of 10 years, from December 4 2012 to December 30 2022. The data source for this research was obtained from historical data on derivative gold prices on the website investing.com.

This study uses 4 research technical stages, namely as follows:

Application of the Geometric Brownian Motion Model

The technical steps taken at this stage are as follows:

- 1) Data grouping. At this stage, historical data on gold derivative prices are grouped based on the number of periods.
- 2) Determine the in-sample data. Data in sample is determined as much n data, which will then be used to build the model.
- 3) Determine the data out sample. Data out sample is determined as much m , used to validate the model.
- 4) Calculating the value of return on assets with data in sample.
- 5) Performing the data normality test in sample return on assets using the Kolmogorov-Smirnov test.
- 6) Calculating the expected value of the mean asset price parameter ($\hat{\mu}$), variance ($\hat{\sigma}^2$), and volatility ($\hat{\sigma}$) from the in sample return data obtained
- 7) Modeling and predicting the price of gold derivatives using the Geometric Brownian Motion method.
- 8) Validating the model using out sample data and calculating the error value of gold derivative prediction prices using the Mean Absolute Percentage Error (MAPE) method.

- 9) Analyze the results of error calculations using *Mean Absolute Percentage Error* (MAPE).
- 10) Repeat the steps in points 4-7 to predict the price of gold derivatives in the 2023 period.

Trend Pattern Analysis Using the Simple Moving Average Method

The technical steps taken at this stage are as follows:

- 1) Group data. At this stage the data is grouped based on a period of 1 month.
- 2) Calculating the moving average value using equation (2.18).
- 3) Analyze trend patterns based on Table 2.3.

Seasonality analysis

The technical steps taken at this stage are as follows:

- 1) Grouping data from trend pattern analysis.
- 2) Analyze the probability of a trend pattern repeating.
- 3) Analyze the probability of the 2023 gold price prediction trend pattern.

RESULT AND DISCUSSION

Price prediction using Geometric Brownian Motion

To get the GBM model solution, assume a function $F(P_t, t) = \ln P_t$. Using the theorem $It\hat{o}$ in equation (2.9), then the following equation is obtained:

$$\begin{aligned} d(\ln P_t) &= \left(\frac{\partial(\ln P_t)}{\partial P_t} \mu P_t + \frac{\partial(\ln P_t)}{\partial t} + \frac{1}{2} \frac{\partial^2(\ln P_t)}{\partial P_t^2} \sigma^2 P_t^2 \right) dt + \left(\frac{\partial(\ln P_t)}{\partial P_t} \sigma P_t \right) dW_t \\ d(\ln P_t) &= \left(\frac{1}{P_t} \mu P_t + 0 + \frac{1}{2} \left(-\frac{1}{P_t^2} \right) \sigma^2 P_t^2 \right) dt + \left(\frac{1}{P_t} \sigma P_t \right) dW_t \\ d(\ln P_t) &= \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t \end{aligned}$$

Then integrate by giving a limit $T[0, t]$, obtained:

$$\begin{aligned} \int_0^t d(\ln P_t) &= \int_0^t \left(\mu - \frac{1}{2} \sigma^2 \right) dt + \int_0^t \sigma dW_t \\ \ln P_t - \ln P_0 &= \left(\mu - \frac{1}{2} \sigma^2 \right) (t - 0) + \sigma (W_t - W_0), W_0 = 0 \\ \ln P_t - \ln P_0 &= \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t \\ \ln \left(\frac{P_t}{P_0} \right) &= \left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t \\ e^{\ln \left(\frac{P_t}{P_0} \right)} &= e^{\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t} \\ \frac{P_t}{P_0} &= e^{\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t} \\ P_t &= P_0 e^{\left(\mu - \frac{1}{2} \sigma^2 \right) t + \sigma W_t}, W_t = \varepsilon \sqrt{\Delta t} \end{aligned} \tag{2.10}$$

So for the difference in the time period limits $p_0 < p_1 < p_2 < \dots < p_n$, the following solutions are obtained :

$$P_t = P_{t-1} e^{\left(\mu - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma \varepsilon \sqrt{\Delta t}} \tag{2.11}$$

So that for each gold derivative price prediction at the time t can be obtained from the GBM model solution as follows:

$$S_t = S_{t-1} \exp\left(\left(\mu - \frac{1}{2}\sigma^2\right)\Delta t + \sigma\varepsilon\sqrt{\Delta t}\right) \quad (2.12)$$

with:

- S_t : Predict the price of gold derivatives at a point in time t
- S_{t-1} : Predict the price of gold derivatives at a point in time $t - 1$
- μ : Drift values
- σ : Value volatility

In-sample data is used to build a gold price model, while out-sample data is used to validate the model. There are no standard rules regarding the comparison of the amount of in-sample and out-sample data used for predictive research, this is adjusted to the character and objectives of the research. Like the research conducted by Moniaga (2011) using a comparison of 70% in-sample data and 30% out-sample data, besides that another study conducted by Sari, et al (2020) used a comparison of 50% in-sample data and 50% out-of-sample data. out sample.

At this stage a prediction of gold derivative prices for a period of 1 year will be made using the Geometric Brownian Motion model to see its feasibility in predicting gold derivative prices, which are presented in the following table:

Table 3. In-Sample and Out-Sample data grouping.

Data Name	Period Year		Comparison (year)
	<i>in Sample</i>	<i>Out Sample</i>	
N1	2021	2022	1:1
N2	2020-2021	2022	2:1
N3	2019-2021	2022	3:1

Grouping is based on the number of periods, with the aim of seeing the amount of influence the data has on the prediction accuracy of research conducted with the assumption that the data is normally distributed. Data N1 uses an in sample and out sample ratio of 1:1, namely 2021 for in sample and out sample in 2022. Data group N2 uses a 2:1 ratio with 2 years of in sample data, namely 2020-2021. The N3 data group uses a data group of 3 years, namely 2019 to 2021.

the normality test of the in sample return data on derivative gold prices using the Kolmogorov-Smirnov is as follows:

- Hypothesis: H_0 : Data in sample returns are normally distributed.
- H_1 : Data in sample return is not normally distributed.

Significance Level : $\alpha = 5\%$

The calculation results are obtained as presented in the following table:

Table 4. Normality test results

Data Name	Means	Standard Deviation	Amount of data	D_{maks}	$D_{\alpha,n}$	Conclusion
N1	0.000545	0.006662	245	0.037786	0.086887	Normal
N2	0.000959	0.007613	491	0.035651	0.061376	Normal
N3	0.000926	0.007005	738	0.027264	0.050062	Normal

Based on the table it can be concluded that, all data H1, H2, and H3 are normally distributed and accepted, because the value with so that data H1, H2, and H3 can be continued for parameter estimation of the Geometric Brownian Motion model. $H_0 D_{maks} < D_{\alpha,n} \alpha = 0,05$ Parameters in the Geometric Brownian Motion gold price model include return expectation values (μ), return variances (σ^2), and volatility values (σ). With the help of the Microsoft Excel application program, the results are summarized in the following table

Table 5. Model parameter estimation.

Data Name	n	$\hat{\mu}$	$\hat{\sigma}^2$	$\hat{\sigma}$
N1	245	0.000545	0.006662	0.000044389
N2	491	0.000959	0.007613	0.00005796
N3	738	0.000926	0.007005	0.00004907

The estimated mean return, variance, and volatility will be the parameters in constructing the Geometric Brownian Motion model for derivative gold prices.

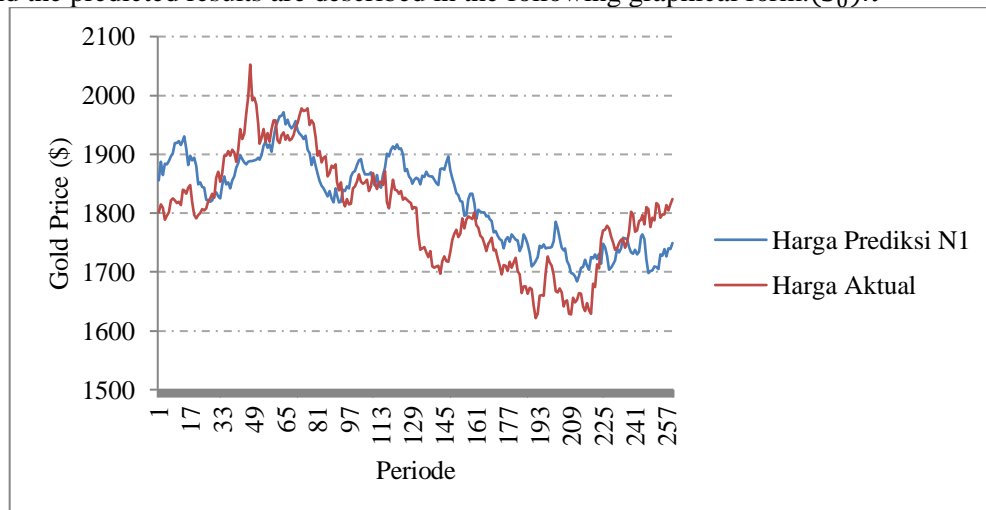
Based on Equation (2.12) with the model parameters of mean return, variance, and volatility in Table 4.3, the derivative gold price model with Geometric Brownian Motion is as follows:

$$N1 \rightarrow S_t = S_{t-1} \exp \left(\left((0,000545) - \frac{1}{2}(0,006662) \right) 1 + (0,000044389)\varepsilon\sqrt{1} \right)$$

$$N2 \rightarrow S_t = S_{t-1} \exp \left(\left((0,000959) - \frac{1}{2}(0,007613) \right) 1 + (0,00005796)\varepsilon\sqrt{1} \right)$$

$$N3 \rightarrow S_t = S_{t-1} \exp \left(\left((0,000926) - \frac{1}{2}(0,007005) \right) 1 + (0,00004907)\varepsilon\sqrt{1} \right)$$

The initial price starts at 1,828.39 with random random variables generated as many times and the predicted results are described in the following graphical form. $(S_0)n$



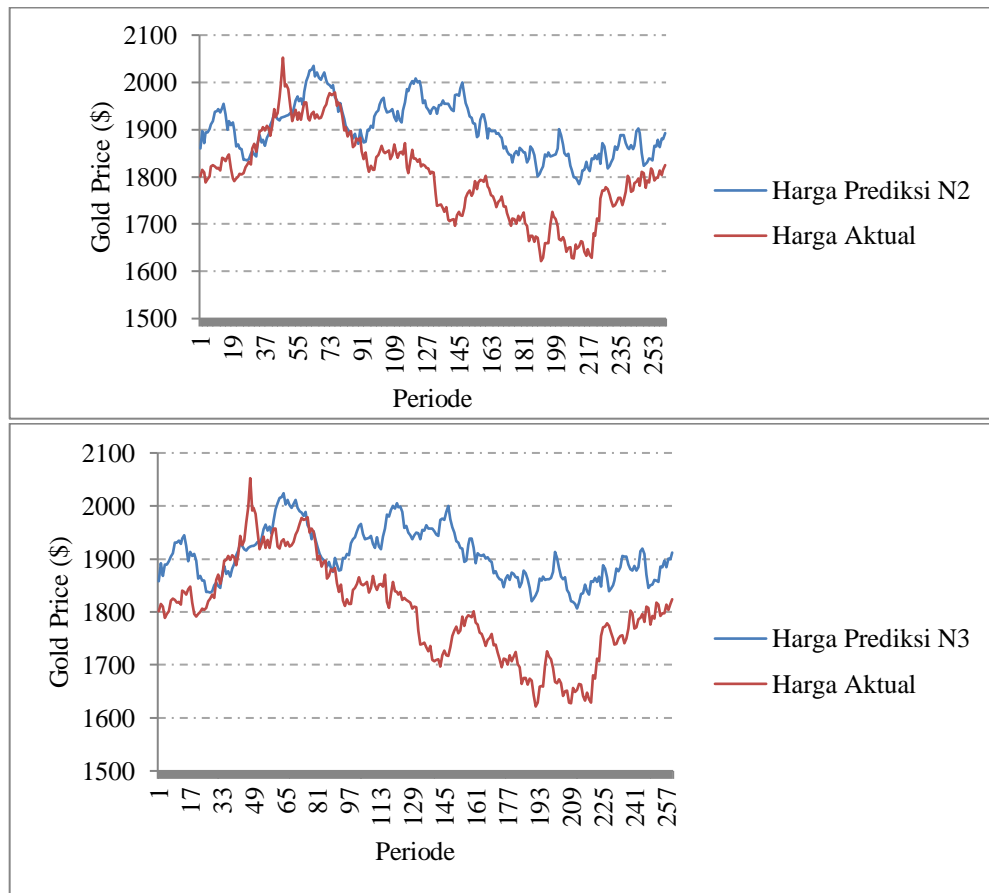


Figure 1. Comparison graph of actual prices (Out Sample) with predicted prices (In Sample) for each data grouping N1, N2, and N

The chart shows that the predicted results of gold derivatives prices from all three groups of data have different variations. The N1 data group with a 1:1 comparison of in-sample and out-sample data shows prediction price movements that are closer to actual price movements. The N2 data group with a ratio of 1: 2 shows a movement density that is not very good compared to the N1 data. The N3 data group shows a price density no better than N1 and N2. However, in general, the predicted movement of each data grouping almost follows its actual price movement.

Using the help of the Microsoft Excel application program, the results of the MAPE test calculation are summarized in the following table:

Table 6. The results of the accuracy test use the Mean Absolute Percentage Error

Data Name	MAPE	%
N1	0.0312489	3.12%
N2	0.05994124	5.99%
N3	0.063559	6.36%

From the table above, it can be concluded that from the prediction accuracy test with out-sample data, MAPE values were obtained for all data groups that showed very good prediction accuracy, because the MAPE values obtained from all data groups, N1, N2, N3.<10%. This indicates that the Brownian Motion Geometric model can be used to predict the price of gold derivatives. To predict the price of gold derivatives in a period of 1 year, the results of the MAPE test show a correlation between the comparison of the amount of

data in sample and out sample, where the N1 data group shows better accuracy with a comparison of the actual amount of data in the sample and out sample by 1: 1 compared to N2 and N3 with a larger comparison in the data in the sample.

Based on the MAPE test analysis that was carried out in the previous step, the prediction of gold derivative prices in 2023 within a period of 1 year with n as many as 260 predictive data will use historical data in 2022. Starting from January 3, 2022 to December 30, 2022.

The return value is tested for normality with H_0 accepted because the value of D -max.< D -t. with $\alpha = 0.05$, it can be concluded that the data in sample return is normally distributed and can be continued for estimation of model parameters.

Parameter estimation is obtained with a return expectation value ($\hat{\mu}$) of -0.00014016, return volatility ($\hat{\sigma}^2$) of 0.0082134, and a variance value ($\hat{\sigma}$) of 0.00006746. Based on equation (2.12), the geometric Brownian Motion model for the predicted price of gold derivatives in 2023 is obtained as follows:

$$S_t = S_{t-1} \exp \left(\left((0,00014016) - \frac{1}{2} (0,0082134) \right) 1 + (0,00006746) \varepsilon \sqrt{1} \right)$$

From this model, predictions for the price of gold derivatives in 2023 are obtained as summarized in the following graph:

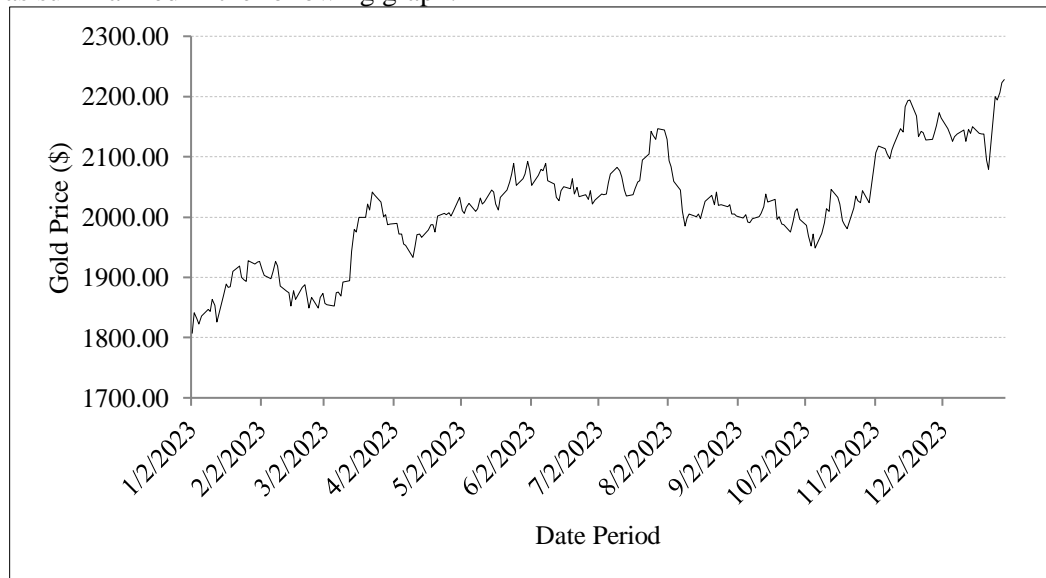


Figure 2. Prediction fic of gold derivatives prices in 2023 using the Geometric Brownian Motion model

Analysis of Trend Movement Patterns using the Simple Moving Average method

Derivative gold prices are grouped into monthly periods using historical gold derivative price data for 10 years, starting from January 1, 2013 to December 30, 2022 as well as the results of the gold derivative price predictions for 2023 obtained in the previous step.

Table 7. grouping of monthly historical data on gold derivative prices.

January		February		...	December	
Date	Price	Date	Price		Date	Price

01/01/2013	1675,15	01/01/2014	1205.90	03/01/2022	1800.85
02/01/2013	1686.75	02/01/2014	1224.89	04/01/2022	1814.56
03/01/2013	1663.35	03/01/2014	1236.55	05/01/2022	1809.53
04/01/2013	1656.85	06/01/2014	1238.05	06/01/2022	1788,68
...
30/01/2013	1677,10	01/30/2014	1243.70	12/01/2022	1825,32
01/31/2013	1663,60	01/31/2014	1243.80		

At this stage, the calculation of the average moving value is carried out using the Simple Moving Average method. This calculation is carried out with the aim of smoothing out the fluctuations in the movement of the trend within a range of monthly periods with a value of 20. *n*After getting the results of the calculation of the moving average, at this stage an analysis of the trend movement pattern is carried out. Drawing conclusions from the analysis of trend movement patterns is based on references in Table 2.3. which is depicted in the plot as follows.

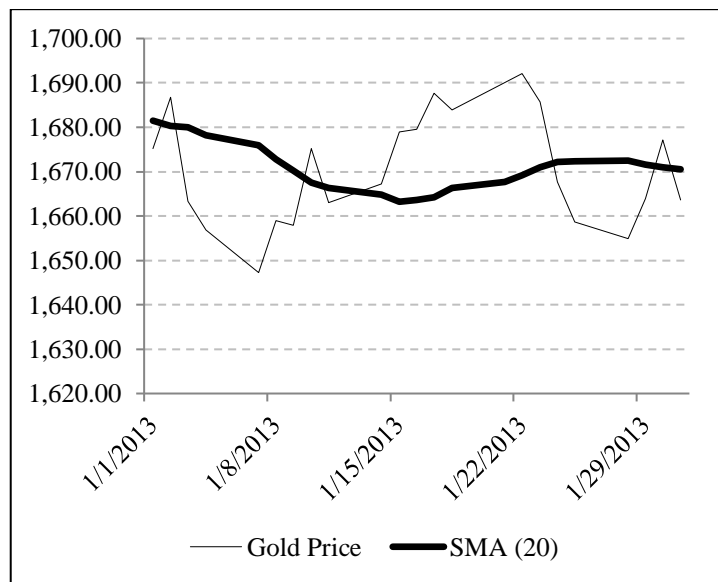


Figure 3. Trend movement pattern in January 2013

From the movement in January 2013 it can be seen that the price of gold derivatives moved up and down crossing the SMA (20) line. So it can be concluded that the price movement pattern in January 2013 was Ranging.

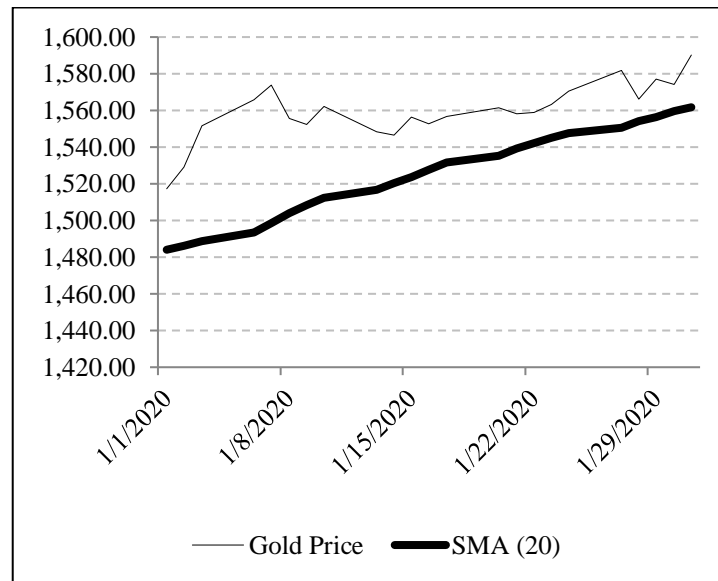


Figure 4. Trend movement pattern for January 2020

From the movement in January 2020, it can be seen that the price of gold derivatives is moving above the SMA (20) line. So it can be concluded that the type of pattern for price movement in January 2020 is a trend movement.

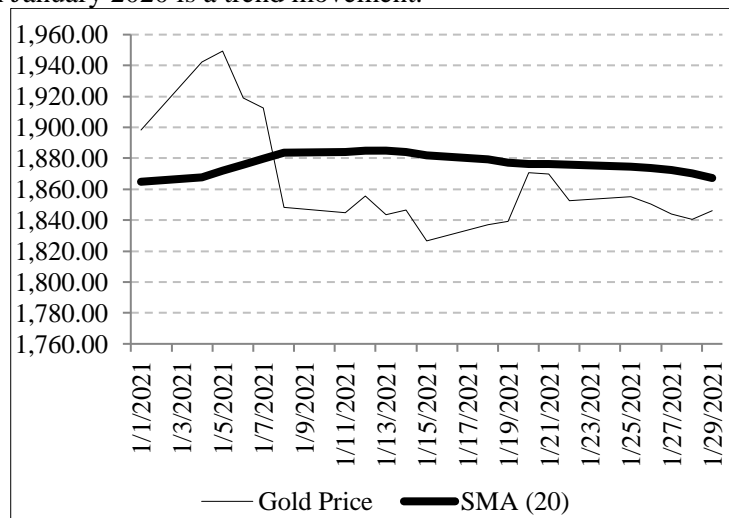


Figure 5. January 2021 trend movement pattern

From the movement in January 2021, it can be seen that the price of gold derivatives has moved to cross the SMA (20) line from top to bottom. So it can be concluded that the type of price movement pattern in January 2021 is a Reversal movement.

Personality analysis

Seasonality analysis is carried out to see seasonal movements, or patterns of trend movements that occur repeatedly. From the results of the analysis of the trend movement pattern in the previous step, the probability of a repetition of the movement pattern for each monthly period within 10 years is determined, and the results are summarized in the following table:

Table 8. The results of the repeatability analysis of gold derivative monthly trend patterns over a 10-year period

Period	Year										T: RA: RE
	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	
Jan	RA	Q	Q	Q	Q	Q	Q	Q	RE	RE	7:1:2
Feb	Q	Q	Q	Q	Q	RA	Q	Q	Q	RE	8:1:1
Mar	RE	RE	RE	RE	RE	RE	RE	RA	Q	RA	1:2:7
April	Q	RA	RA	RA	Q	RE	Q	Q	Q	RA	5:4:1
May	Q	RA	RA	RE	RE	Q	RA	RA	Q	Q	4:4:2
June	Q	RE	RA	Q	RE	Q	Q	RE	RE	RA	4:2:4
July	RE	RE	Q	RA	RE	Q	Q	Q	RE	Q	5:1:4
august	Q	RA	RE	RE	Q	Q	Q	RA	RE	RE	4:2:4
Sept	RE	Q	RA	RA	RE	RA	RE	Q	RE	Q	3:3:4
Oct	RE	RA	Q	Q	RA	RE	RA	RA	RE	RE	2:4:4
Nov	Q	RE	Q	RE	RA	RA	RE	RE	RE	Q	3:2:5
dec	Q	RE	RA	Q	RE	Q	Q	RA	RE	Q	5:2:3

1. Information :

T = Trends

RA = Ranging

RE = Reversal

From the data on price movement patterns of gold derivatives in Table 4.6, the seasonal probabilities of price movements of gold derivatives over a 10-year period are obtained which are summarized in Table 9 below.

Table 9. The probability of repeating the movement of the gold derivative price trend pattern for 10 periods.

Period	Trends	Rank	Reversal	2023 predictions
January	70%	10%	20%	Trends
February	80%	10%	10%	Reversal
March	10%	20%	70%	Trends
April	50%	40%	10%	Reversal
May	40%	40%	20%	Trends
June	40%	20%	40%	Reversal
July	50%	10%	40%	Trends
August	50%	25%	25%	Trends
September	30%	30%	40%	Rank
October	20%	40%	50%	Reversal
November	30%	20%	50%	Trends
December	50%	20%	30%	Trends

Based on the table, in 2023 there is a possibility of gold derivative price movements following trends, reversals and ranging with different probabilities for each month. This probability is based on probability data for the seasonality of gold derivative price movements over the previous 10 years.

The results of the seasonality analysis show that January has the best level of probability and prediction for making transactions with a probability of predicting the trend pattern movement of 70%. Meanwhile, February, March and April show the lowest probability of predicting the movement of the gold derivative pattern with a probability of 10%.

CONCLUSION

Based on the problems raised in this study, it can be concluded that by grouping 3 variations in the comparison of the number of n In Sample and Out Sample, a Derivative

Gold Price Brownian Motion Geometry model is obtained with a prediction error value calculated using the Mean Absolute Percentage Error (MAPE) each of 3.12%, 5.99% and 6.36%. It indicates that the prediction accuracy of gold derivative prices using the Geometric Brownian Motion model is very good. So it can be concluded that the Geometric Brownian Motion method can be used to predict the price of gold derivatives. Of the three variations of the ratio of n In Sample and Out Sample, the N1 variation shows better accuracy with a ratio of 1:1 compared to the other 2 variations.

The results of the seasonality analysis obtained the probability of repetition of each pattern of monthly gold derivative price movements for 10 years. It is indicated that the price movement pattern of gold derivatives that occurs in the prediction for 2023 will be correlated according to the probability level of the results of the analysis. The results of the seasonality analysis show that January has the best level of probability and prediction for making transactions with a probability of predicting the trend pattern movement of 70%. Meanwhile, February, March and April show the lowest probability of predicting the movement of the gold derivative pattern with a probability of 10%.

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